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Finite Size Scaling and Stability of Matter in Super-intense Laser Fields

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Phase Transitions

- **Classical: Classical phase transitions are driven by thermal energy fluctuations**

  Like the melting of an ice cube:

  Solid → Liquid → Gas

- **Quantum: Quantum phase transitions, at T=0, are driven by the Heisenberg uncertainty principle**

  Like the melting of a Wigner crystal: Two dimensional electron layer trapped in a quantum well

  Insulating State → Metallic Conducting State

Quantum Phase Transitions

- Transitions that take place at the absolute zero of temperature, T=0, where crossing the phase boundary means that the quantum ground state energy $E_0(\lambda)$ of the system changes in some fundamental way.
- This is accomplished by changing some parameter in the Hamiltonian of the system $H_0(\lambda)$.
- We shall identify any point of non-analyticity in the ground state energy $\lambda = \lambda_C$, as a quantum phase transition.

Finite Size Scaling

In statistical mechanics, the finite size scaling method provides a systematic way to extrapolate information obtained from a finite system to the thermodynamic limit.

Importance:

The existence of phase transitions is associated with singularities of the free energy. These singularities occur only in the thermodynamic limit.

Yang and Lee (1952)

In the present approach, the finite size corresponds not to the spatial dimension, as in statistics, but to the number of elements in a complete basis set used to expand the exact eigenfunction of a given Hamiltonian.

$$\psi = \sum_{n=0}^{\infty} a_n \phi_n \cong \sum_{n=0}^{M} a_n \phi_n$$

(Variational Calculations)

Classical $(N \to \infty)$ ⇔ Quantum $(M \to \infty)$

Finite Size Scaling: Quantum Mechanics

$$H = H_0 + V_\lambda$$

$$\psi_\lambda^{(N)} = \sum_n^{M(N)} a_n^{(N)}(\lambda)\phi_n$$

Quantum Mechanics

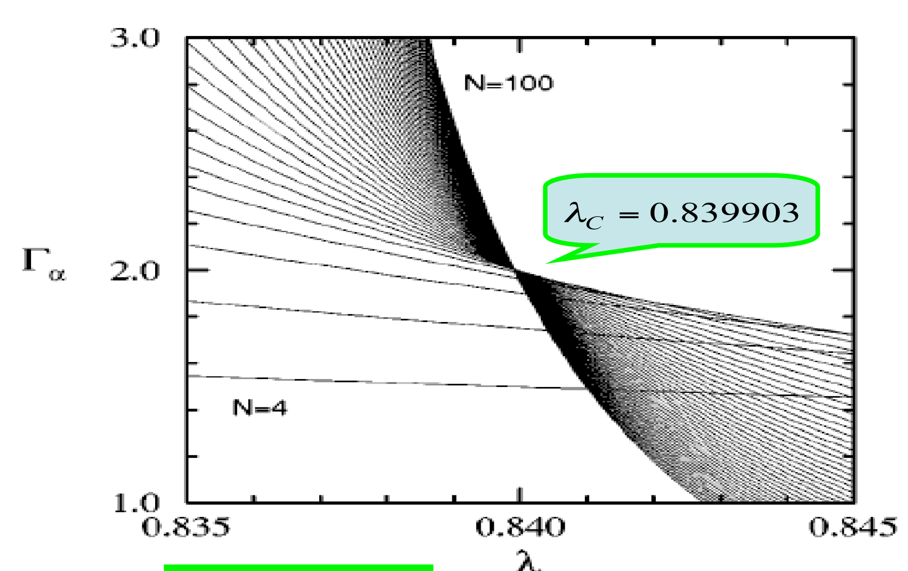
The FSS ansatz

$$\langle O \rangle_\lambda^{(N)} \sim \langle O \rangle_\lambda F_O\left(N|\lambda - \lambda_C|^\nu\right)$$

$$\Delta_O(\lambda; N, N') = \frac{\ln\left(\langle O \rangle_\lambda^{(N)} / \langle O \rangle_\lambda^{(N')}\right)}{\ln(N'/N)}$$

The curves intersect at the critical point

$$\Delta_O(\lambda_C; N, N') = \Delta_O(\lambda_C; N'', N)$$



$$\Gamma_\alpha = \frac{\Delta_H}{\Delta_H - \Delta_{\frac{\partial V}{\partial \lambda}}} \qquad \lambda_c^{exact} = 0.839908$$

High Frequency Fluent Theory (HFFT)

Hamiltonian of atomic system in super-intense laser fields

H = *kinetic term* + *coulomb term* + *laser term* + *e-e term*

$$H = \sum_{i=1}^{N}\left\{\frac{1}{2}p_i^2 + V_0(r_i, a_0) + \sum_{j=1}^{i-1}\frac{1}{|r_i - r_j|}\right\}$$

V_0 is the "dressed" Coulomb potential. (Coulomb potential in laser fields)

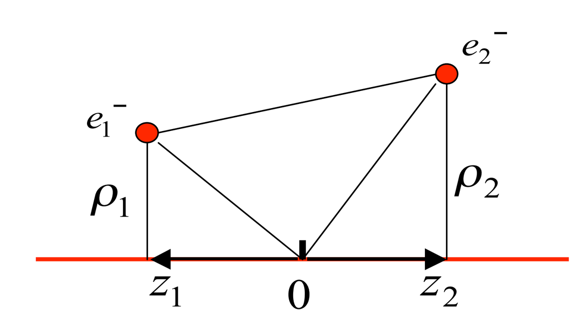
$$V_0(a_0, r)_{\gamma_r} = \frac{1}{2\pi}\int_0^{2\pi}\frac{d\phi}{\left|\vec{r} + a_0(\hat{e}_x\cos\phi + \tan\delta\,\hat{e}_y\sin\phi)\right|}$$

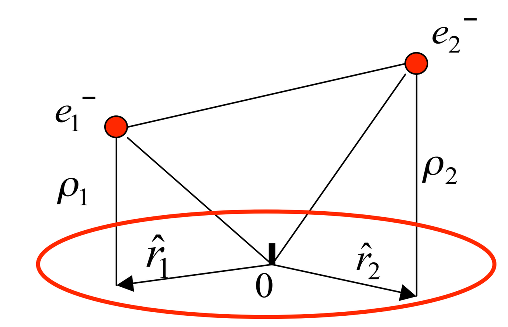
$\delta = 0$ corresponds to linear polarization,

$\delta = \pm\pi/4$ corresponds to circular polarization.

$\alpha_0 = E_0/\omega^2$, where E_0 and ω are the amplitude and frequency of the laser field.

Large D stability in super-intense linearly and circularly polarized laser fields



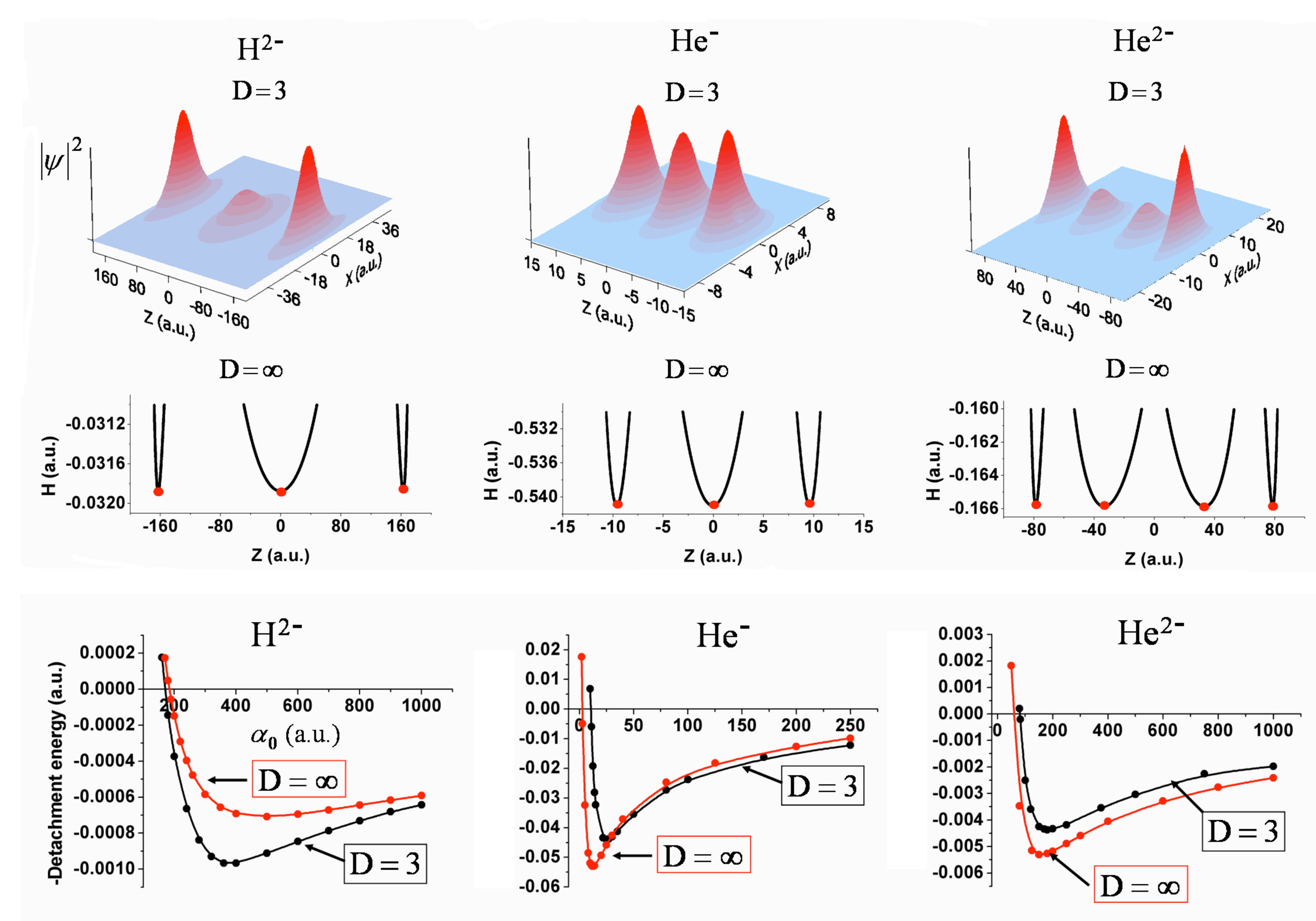


$$H = \frac{1}{2}\sum_{i=1}^{N}\frac{1}{\rho_i^2} + \sum_{i=1}^{N}V_0(\rho_i, z_i) + \sum_{i=1}^{N}\sum_{j=i+1}^{N}\frac{1}{\sqrt{\rho_i^2 + \rho_j^2 + (z_i - z_j)^2}}$$

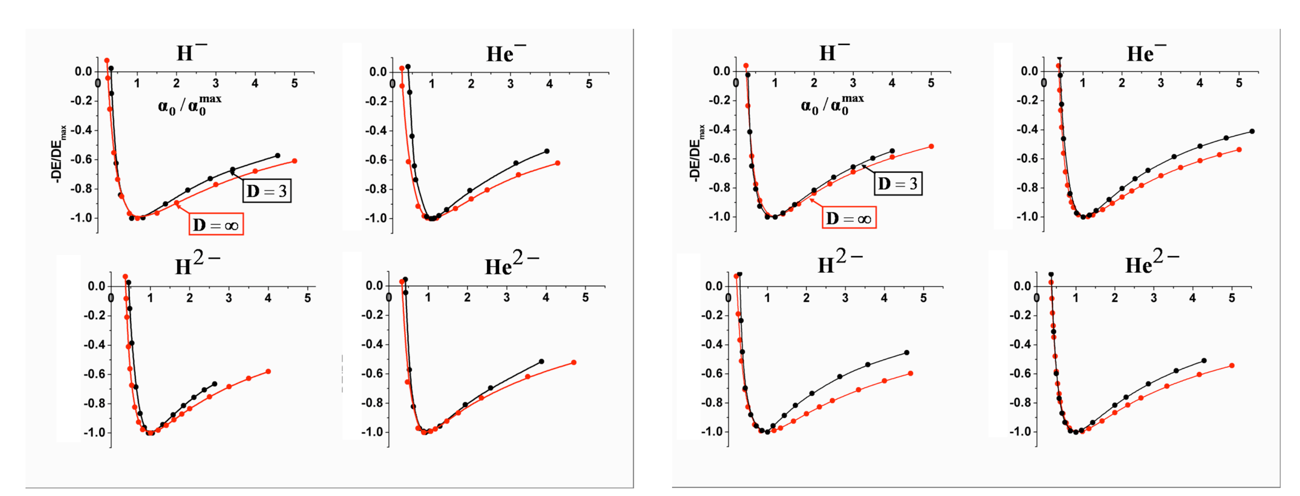
$$V_0(\rho_i, z_i) = \frac{-Z}{2\pi}\int_0^{2\pi}\frac{d\phi}{\sqrt{\rho_i^2 + (z_i + \alpha_0 Sin\phi)^2}}$$

$$H = \frac{1}{2}\sum_{i=1}^{N}\frac{1}{\rho_i^2} + \sum_{i=1}^{N}V_0(\rho_i, \hat{r}_i) + \sum_{i=1}^{N}\sum_{j=i+1}^{N}\frac{1}{\sqrt{\rho_i^2 + \rho_j^2 + (\hat{r}_i - \hat{r}_j)^2}}$$

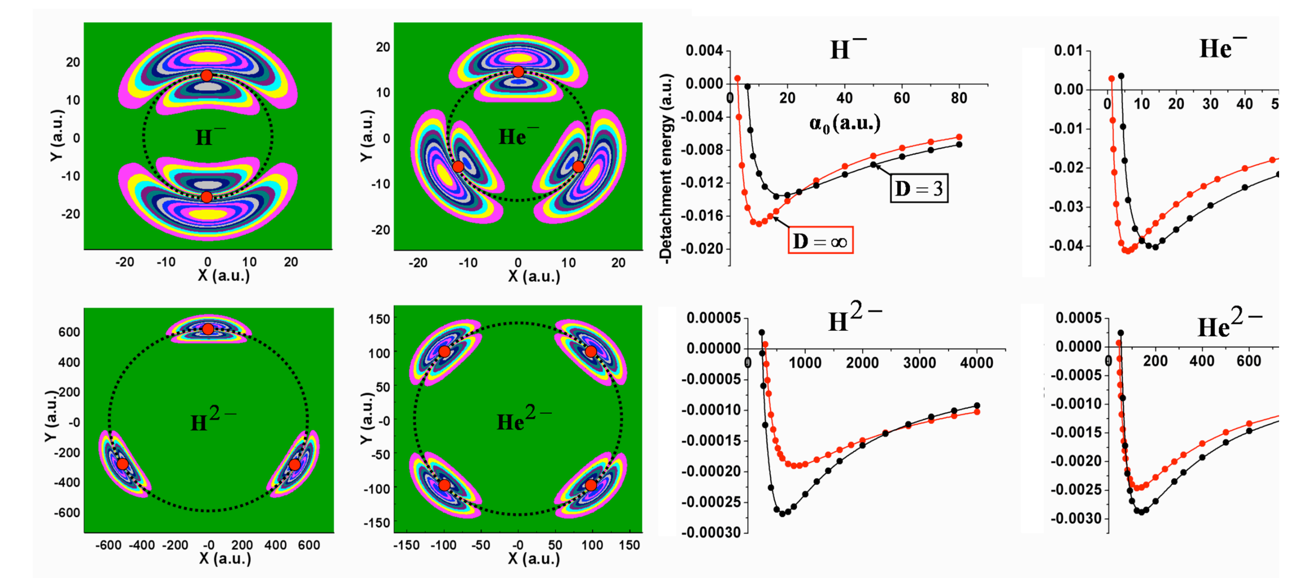
$$V_0(\rho_i, \hat{r}_i) = \frac{-Z}{2\pi}\int_0^{2\pi}\frac{d\phi}{\sqrt{\rho_i^2 + (\hat{r}_i\cdot\hat{e}_x + \alpha_0 Cos\phi)^2 + (\hat{r}_i\cdot\hat{e}_y + \alpha_0 Sin\phi)^2}}$$



Top: Ground state wave function of He^-, H^{2-} and He^{2-} in a linearly polarized high-frequency super-intense laser field. **Middle**: Electric distribution for large dimension. **Bottom**: Negative of detachment energy of ground state as a function of α_0 for both 3 dimension and large dimension.



Scaled negative of detachment energy of ground state as a function of α_0 for H^-, He^-, H^{2-} and He^{2-}. (left for linear and right for circular polarization).



Left: Electric distribution of round state wave function of H^-, He^-, H^{2-} and He^{2-} in a circularly polarized high-frequency super-intense laser field. **Right**: Negative of detachm energy of ground state as a function of α_0 for both 3 dimension and large dimension.

Summary

- **Symmetry breaking of electronic structure configurations resemble classical phase transitions.**
- **Finite Size Scaling method can be used to obtain critical parameters for the quantum Hamiltonian: Critical charges, critical dipole and quadrupole-bounc anions,…**
- **Quantum phase transitions can be used to explain and predict the stability of atoms, molecules and quantum dots.**
- **Multiply charged negative atomic anions can be stabilized in super-intense laser fields.**